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An Exploration Of Certain Properties Of Chaotic Dynamics In G-Spaces

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Abstract. Let X be a topological G −space and let $F_n(X)$ be the $n-fold$ symmetric product of X, for any positive integer n. Let $f : X \to X$ be a function, we consider the induced functions $F_n(f)$: $F_n(X) \to F_n(X)$. In this paper we presented some chaotic properties like: *Transitive, Mixing, Weakly Mixing, Totally Transitive, Exact, Strongly Transitive and Chaotic.* We investigate the relationship between systems f and $F_n(f)$ for the above properties.

الملخص. ليكن الفضاء التبولوجي **X** هو فضاء – جي, ولتكن $F_n(X)$ فضاء الضرب المتماثل ذي الرتبة **n** للفضاء **X** , ولأي عدد **n** صحيح و موجب. لتكن $f : X \to X$ دالة سنتضمن دوال محتثة بالشكل : $F_n(f)$: $F_n(X) \to F_n(X)$. في هذه الورقة البحثية سنعرض وندرس بعض الخواص الفوضوية مثل : التعدي, المزج, المزج الضعيف, التعدي الكلي, و الفوضوية وغيررها. سندرس العلاقة لهذه الخوص الفوضوية بين الدالة f والدالة $F_n(f)$.

Keywords: G −space, dynamical systeems, topological transitivity, symmetric products, indced functions $F_n\ (f)$

1. Introduction

The study of dynamical systems in modern mathematical fields has captivated numerous mathematicians. A dynamical system consists of multiple states linked by rules or conditions that dictate the current state based on preceding states. In topological dynamical systems, there are two classical types: discrete and continuous dynamical systems. Recently, numerous intriguing studies have been done on discrete dynamical systems of the form $z_{i+1} = f(z_i)$, $i = 0, 1, 2, \ldots$, and f is a continuous self-map. A specific category of dynamical systems, known as chaotic dynamical systems, has been thoroughly investigated. One of the most active areas of mathematics is chaos theory, which has drawn many mathematicians because of its intriguing applications in many different kinds of fields, including physics, economics, and biology. A complex system will be made simpler and more predictable by understanding this notion. Numerous scientists define chaos theory according to their own definitions as (Devaney, Li-Yorke, Gualic, Wiggin, etc.). However, Devaney's concept of chaos is the most widely accepted and strong. Chaos theory offers whole new characteristics and animated maps from the standpoint of the dynamical systems concepts. Chaos theory has been studied in a variety of contexts outside of this space domain. Group action, set-valued mappings, iterated function systems, and other types of chaos have recently been developed and researched.

G-space is the topic of this study, in the field of topological dynamical systems, offers a framework for investigating the dynamics of a group's action on a topological space. By investigating G-space, researchers can gain insights into the behavior and properties of dynamical systems *under group actions, leading to a deeper understanding of the behavior and properties of dynamical systems under group actions, leading to a deeper understanding of the interaction between topology and dynamics in various contexts. Furthermore, research into G-space can help discover new phenomena, disclose connections between many different areas of mathematics, and lead to the establishment of theoretical foundations for applications in fields such as physics, engineering, and biology. In recent years, an increasing number of studies have turned their focus to the study of G-space. In [4], R. Das and T. Das studied transitivity in $G-$space. The condition for the limit function to be topologically $G-$transitive is also provided. They identified the conditions that are required for a sequence's limit function of $G-$ transitive map to be $G-$ transitive. In [2], R. Das defined the $G-$transitive subset for a continuous map on a compact metric space. She demonstrated that a $G-$transitive subset of a sequence of continuous maps (f_n) is also a $G-$transitive subset of the limit map (f). In [3], R. Das investigated and explored the definitions of several chaotic notation for sequence mappings in metric G-space in two ways (iterative and successive), such as G-periodic point, G-transitive, G-SDIC, and G-chaotic. She additionally presented some instances of maps with G-chaotic sequence. In [5], R. Das investigated and discussed sufficient conditions when two maps are G-chaotic and their product is G-chaotic, as well as the product of two $G-mixing$ maps is $G-mixing$. In [13], M. Abbas and I. ALshara'a presented some results and generalized the definitions of locally eventually onto, weakly blending, strongly blending, and touhey property maps on G-space. They investigated the product maps of blending (strongly and weakly) maps in G-space, as well as the relationships between strongly blending and the Touhey property with other G-space concepts. In [8], M. Garge and R.Das presented and investigated certain chaotic properties on G-space that are stronger than forms of transitivity, such as totally G-transitive, weakly G-mixing, strongly G-mixing, and G-*

minimal. They proved a few results that showed the relationship between these concepts. In [9], M.Garg and R.Das investigated and explored G-transitivity and G-minimality maps using the notation G-regular periodic decomposition on G-space. In [12], M. Abass and I. AL-Shara'a define and study the convergence of several chaotic properties on G-space, as well as prove some of these properties (minimal, blending, and mixing) of a sequence map and product on G. In [15], I.J. Kadhim and S.K. Jabur investigated various dynamical notions in G-space and analyzed the notation for Devaney's G-chaotic. They also defined and verified equicontinuous maps on G-space. In [10], M. Garg and R.Das presented several types of map transitivity on G-space, such as positive, infinite, orbital, G-ω-transitive, and G-transitive point. These concepts were thoroughly investigated, including the necessary conditions for orbit G-transitive to imply G-ω-transitive.

In [25], K. YAN, Q. LIU, and F. ZENG investigated various topological notions for group actions. They defined the concept of scattering, mild mixing, and other concepts for group action, and they proved that a completely G-transitive with a dense collection of G-periodic points is weakly G-mixed.

Historically, the development of hyperspace theory had its beginnings in the early to twentieth centyry, with the stydies of F.Hausdorff and L.Vietoris. The most studied hyperspaces of a compact metric space X are: the hyperspace 2^x which consists of all the nonempty compact subsets of X; given a natural number n, the hyperspace G_n (X) consisting of the elements of 2^x that have the most n components (the n-fold hyperspace of X); and the hyperspace H_n (X) formed by the elements of 2^x which have at most n points (the n-fold symmetric product of X). Each of these hyperspaces is endowed with the topology induced by the Hausdorff measure. These hyperspaces have been deeply studied in continuum theory [6], [20], and [21].

One line of research focuses on anlayzing the relationships between the dynamical system (x, f) and the othere dynamical systems ($(2^x, 2^f)$, $(\mathcal{G}_n(X), \mathcal{G}_n(f))$ and ($F_n(X)$, $F_n(f)$). It is clear that research on hyperspace dynamics has received increased attention in recent years, as seen [1], [7], [16], [17], [18], [19], [22], [23], and [24].

Several topological features of symmetric products are investigated, including transitivity, chaotic mixing, weakly mixing, strongly transitivity, and totally transitivity

Our work focuses on the link between dynamical systems (X, f) and ($F_n(X)$, $F_n(f)$), where X is G −space. We present the definitions and preliminary information needed for section 3 in section 2. Using the concepts mentioned above, we examine some theorems in section 3 that prove the relationship between the systems (X, f) and ($F_n(X)$, $F_n(f)$). We give a summary of our results in section 4.

2. Basic Definitions

Let G be a finitely generated topological group. Let X be the topological space induced by the matric space. This section introduces numerous definitions that will be used to construct our results and properties in G −space.

Definition **2.1** [14]

Let X be a Topological space, G be a topological group, and $\theta : G \times X \to X$ is a mapping. The triple (X, G, θ) is a metric $G-$ space if it satisfies the following conditions::

- i- $\theta(e, y) = y$, *for all* $y \in X$, *where e is the identity of* G.
- ii- $\theta(g, \theta(s, y)) = \theta(g * s, y)$, *for all* $y \in X$ *and* $g, s \in G$.
- iii- θ *is continuous.*

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X is referred to as a compact metric G −space if it is compact. For $y \in X$, we will denoted by $G_f(y)$ to refers to the G − orbit of y, is given as the set $\{g.f^k(y): g \in G, k \geq 0\}$. The set of all periodic points is denoted by $G - per(f(y))$. For a topological G-space (X,τ) and positive integer n, the $n - fold$ symmetric product of X can be defined as:

$$g.\mathrm{F}_n(X) = \{\, g.D \subseteq X : D \neq \emptyset \text{ and has atmost } n \text{ elemnts}\}$$

Definition **2.2**:

Let (X, τ) be a topological G −space, $f: X \to X$ be a function, and $n \in \mathbb{N}$. The function $\mathrm{F}_n(f): \mathrm{F}_n(X) \to \mathrm{F}_n(X)$ is define us:

$$g.\mathrm{F}_n(f)(D) = g.f(D), \text{for all } D \in g.\mathrm{F}_n(X).$$

The function $\mathrm{F}_n(f)$ is an induced function by f to the hyperspace $\mathrm{F}_n(X)$

Definition **2.3** [4]

Let (X, τ) be a topological G −space and $f: X \to X$ be continuous map. The map f is called G − transitive if for every two non−empty open subsets $\mathcal{V}$ and $\mathcal{U}$ of X, there is $r \in \mathbb{Z}$ such that,

$$g.f^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset, \; g \in G.$$

Definition **2.4** [8]

Let (X,τ) be topological G −space and $f: X \to X$ be continuous map. The map f is totally G −transitive if f^s is G-transitive for every $s > 1$.

Definition **2.5**

Let (X,τ) be topological G −space and $f: X \to X$ be continuous map. The map f is strongly G −transitive if for each non−empty $\mathcal{V}$ open subset of X, there is $r \in \mathbb{N}$ and $g \in G$ such that

$$X = \bigcup_{k=0}^{r} g.f^k(\mathcal{V})$$

Definition **2.6** [8]

Let (X, τ) *be topological* G *–space and* $f: X \to X$ *be continuous map. The map* f *is* G *–mixing if for each two non-empty open subsets* $\mathcal{V}$ *and* $\mathcal{U}$ *of* X*, there is* $m \in \mathbb{N}$ *and* $g \in G$ *such that for each* $k \geq m$ *satisfying*

$$g.f^k(\mathcal{U}) \cap \mathcal{V} \neq \emptyset.$$

Definition **2.7** [8]

Let (X, τ) *be topological* G *–space and* $f: X \to X$ *be continuous map. The map* f *is called weakly* G *–mixing if the Cartesian product* $g.f \times g.f$ *is* $G \times G$*-transitive. In other words, for any open sets* $\mathcal{U}, \mathcal{V}, W$ *and* Y *nonempty of* X*, such that* $\mathcal{U} \times \mathcal{V}$ *and* $W \times Y$ *are open subsets of* $X \times X$ *then there is* $k \in n$ *and* $(p, q) \in G \times G$

$$(g, h)(f \times f)^k (\mathcal{U} \times \mathcal{V}) \cap W \times Y \neq \emptyset .$$

Definition **2.8**

Let (X, τ) *be topological* G *–space and* $f: X \to X$ *be continuous map. The map* f *is* G *–locally everywhere onto (simple* $G - l.e.o$*) if for all nonempty subset* $\mathcal{U}$ *of* X*, there is* $k \in N$ *and* $g \in G$ *such that*

$$g.f^k(\mathcal{U}) = X .$$

Definition **2.9**

Let (X, τ) *be topological* G *–space and* $f: X \to X$ *be continuous map. The map* f *is* G *– chaotic if it is* G *–transitive and* $G - per(f)$ *is dense in* X*.*

3. Some properties and theorems

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Given a finite collection of nonempty subsets $\mathcal{U}_1, \dots, \mathcal{U}_k$ *of* X, *we define us* $\langle \mathcal{U}_1, \dots, \mathcal{U}_k \rangle_n$ *the subset of* $F_n(X)$: $\{D \in F_n(X) : D \subseteq \bigcup_{i=1}^{k} \mathcal{U}_i$ *and* $D \cap \mathcal{U}i \neq \emptyset$, *for each* $i \in \{1, \dots, k\}\}$.

The collection $\mathfrak{B}$, *define by*

$$\mathfrak{B} = \{\langle \mathcal{U}_1, \dots, \mathcal{U}_k \rangle_n, \mathcal{U}_i \in \tau, i = 1, \dots, k \text{ and } k \in \mathbb{N}\},$$

Serves a base genrating a topology on $\mathcal{H}_n(X)$, *this topology is referred to as the Vietoris topology which is denoted by* τ_V. *The the next result, which appearsin* [11, *Lemma* 4.2].

THEOREM 3.1: *Let* (X, τ) *be a topological space and let* $n \in \mathbb{N}$. *Then*

$$\mathfrak{B}' = \{\langle \mathcal{U}_1, \dots, \mathcal{U}_k \rangle_n, \mathcal{U}_i \in \tau, \text{for all } i \in \{1, \dots, k\}\}$$

is a base for the Vietoris topology τ_V *on* $F_n(X)$.

THEOREM 3.2: *Let* X *be a* $G-$ *space, let* $f : X \to X$ *be a function and let* $n \in \mathbb{N}$. *Then* $F_n(f)$ *is* $G-$*mixing if and only if* f *is* $G-$*mixing.*

Proof : *Let as assume that* $F_n(f)$ *is* $G-$*mixing. We will show that* f *is G-mixing, let* $\mathcal{U}$ *and* $\mathcal{V}$ *be subsets of* X *that are open. There are open subsets* $\langle \mathcal{U} \rangle_n$ *and* $\langle \mathcal{V} \rangle_n$ *of* $F_n(X)$. *Since* $F_n(f)$ *is* $G-$*mixing, there is* $\aleph \in \mathbb{N}$ *such that,*

$$g_n.[F_n(f)]^r(\langle \mathcal{U} \rangle_n) \cap \langle \mathcal{V} \rangle_n \neq \emptyset,$$

for each $r \geq \aleph$ *and* $g_n \in G$. *We show that for all* $r \geq \aleph$,

$$g_n.f^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset.$$

Let $r \geq \aleph$, *Since*

$$g_n.[F_n(f)]^r(\langle \mathcal{U} \rangle_n) \cap \langle \mathcal{V} \rangle_n \neq \emptyset, \text{ for all } g_n \in G$$

there is $B \in \langle \mathcal{U} \rangle_n$ *such that*

$$g_n.[F_n(f)]^r(B) \in \langle \mathcal{V} \rangle_n.$$

Thus, $B \subset \mathcal{U}$ *and* $g_n.f^r(B) \subseteq \mathcal{V}$.

Then,

$$g_n.f^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset. \text{ for all } g_n \in G$$

Therefore, f is $G-$mixing.

We then show that $\mathrm{F}_n(f)$ is G-mixing, assuming that f is G-mixing. Let $\mathcal{U}$ and V be nonempty open subsets of $\mathrm{F}_n(X)$. Then, by Theorem (3.1), there is nonempty open subsets $\mathcal{U}_1, \dots, \mathcal{U}_n$, $\mathcal{V}_1, \dots, \mathcal{V}_n$ of X and $g_1, \dots, g_n$ of G such that $\langle \mathcal{U}, \dots, \mathcal{U}_n \rangle_n \subseteq \mathcal{U}$, $\langle \mathcal{V}_1, \dots, \mathcal{V}_n \rangle_n \subseteq \mathcal{V}$ and $\langle g_1, \dots, g_n \rangle_n \subseteq G$. Since f is $G-$mixing, for all $i \in \{1, \dots, n\}$, there is $\aleph_i \in \mathbb{N}$ such that

$$g_i\, f^r(\mathcal{U}_i) \cap \mathcal{V}_i \neq \emptyset, \text{ for all } r \geq \aleph_i \text{ and } g_i \in G.$$

Let $\aleph = \max\{\aleph_1, \dots, \aleph_n\}$. Note that,

$$g_i\, f^r(\mathcal{U}_i) \cap \mathcal{V}_i \neq \emptyset, \text{ for all } r \geq \aleph \text{ and } g_i \in G.$$

Hence, for all $i \in \{1, \dots, n\}$, let $y_i \in \mathcal{U}_i$ such that

$$g_i\,.f^r(y_i) \in \mathcal{V}_i.$$

Let $C = \{y_1, \dots, y_n\}$. Observe that $C \in \langle \mathcal{U}_1, \dots, \mathcal{U}_n \rangle_n$ and

$$g_i\,.[\mathrm{F}_n(f)]^r(C) \in \langle \mathcal{V}_1, \dots, \mathcal{V}_n \rangle_n,\ g_i \in G$$

Thus,

$$g_i\,.[\mathrm{F}_n(f)]^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset,\ g_i \in G$$

Hence, for all $r \geq \aleph$,

$$g_i\,.[\mathrm{F}_n(f)]^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset,\ g_i \in G$$

Therefore, $\mathrm{F}_n(f)$ is $G-$mixing.

□

***THEOREM* 3.3** : *Let X be a $G-$ space, let $f: X \to X$ be a function and let $n \in \mathbb{N}$. Then $\mathrm{F}_n(f)$ is $G-$l.e.o if and only if f is $G-$l.e.o.*

Proof : Let as assume that $\mathrm{F}_n(f)$ is $G-$l.e.o.. We will show that f is $G-$ l.e.o.. Let $\mathcal{U}$ be a nonempty open subset of $\mathrm{F}_n(X)$. Then, by Theorem (3.1), there is $\mathcal{U}_1, \dots, \mathcal{U}_n$ nonempty open subsets of X such that $\langle \mathcal{U}_1, \dots, \mathcal{U}_n \rangle_n \subseteq \mathcal{U}$. Since f is $G-$ exact, there is $r_1, \dots, r_n \in \mathbb{N}$ such that,

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$$g.f^{r_i}(\mathcal{U}_i) = X, \text{ for all } g \in G.$$

Let $r = \max\{r_1, \dots, r_n\}$. *Since* f *is* G *–l.e.o., we conclude that* f *is surjective. Thus,*

$$g.f^{r}(\mathcal{U}_i) = X, \text{ for all } i \in \{1, \dots, n\} \text{ and } g \in G.$$

Now, we show that:

$$g.\mathrm{F}_n(X) \subseteq g.[\mathrm{F}_n(f)]^k(\langle \mathcal{U}_1, \dots, \mathcal{U}_n \rangle_n),\ g \in G.$$

Let $S = \{s_1, \dots, s_r\} \in \mathrm{F}_n(X)$, *with* $t \leq n$. *Define* $C = \{s_1, \dots, s_t, s_{t+1}, \dots, s_n\}$, *where* $s_t = s_{t+1} = \cdots = s_n$. *Then, for all* $i \in \{1, \dots, n\}$,

$$s_i \in X = g.f^k(\mathcal{U}_i),\ g \in G.$$

Thus, for each $i \in \{1, \dots, n\}$, *there is* $\sigma_i \in \mathcal{U}_i$ *such that*

$$g.f^k(\sigma_i) = s_i,\ g \in G.$$

Let $D = \{d_1, \dots, d_n\}$. *Then* $A \in \langle \mathcal{U}_1, \dots, \mathcal{U}_n \rangle_n$ *and*

$$g.[\mathrm{F}_n(f)]^k(D) = C = S.$$

Thus,

$$S \in g.[\mathrm{F}_n(f)]^k(\langle \mathcal{U}_1, \dots, \mathcal{U}n \rangle n).$$

Hence,

$$g.\mathrm{F}_n(X) = g.[\mathrm{F}_n(f)]^k(\langle \mathcal{U}_1, \dots, \mathcal{U}_n \rangle_n).$$

Therefore,

$$g.[\mathrm{F}_n(f)]^k(\mathcal{U}) = g.\mathrm{F}_n(X),\ g \in G.$$

Thus,

$$g.f^k(\mathcal{U}) = X,\ g \in G.$$

so, f *is* G *–l.e.o. .*

□

THEOREM 3.4: *Let* X *be a* G *–space, let* $f : X \to X$ *be a* function *and let* $n \in \mathbb{N}$. *If* $\mathrm{F}_n(f)$ *is* G *–transitive, then* f *is* G *–transitive.*

Proof : Let as assume that $\mathrm{F}_n(f)$ *is* G *– transitive. We will show that* f *is* G *– transitive. Let* $\mathcal{U}$ *and* $\mathcal{V}$ *be nonempty, open subsets of* X. *Then* $\langle \mathcal{U} \rangle_n$ *and* $\langle \mathcal{V} \rangle_n$ *are nonempty open subsets of* $\mathrm{F}_n(X)$. *Since* $\mathrm{F}_n(f)$ *is* G *–transitive, there exists* $r \in \mathbb{N}$ *such that,*

$$g.[\mathrm{F}_n(f)]^r(\langle \mathcal{U} \rangle_n) \cap \langle \mathcal{V} \rangle_n \neq \emptyset,\ g \in G.$$

Thus, there is $D \in \langle \mathcal{U} \rangle_n$ *such that*

$$g.[F_n(f)]^r(D) \in \langle \mathcal{V} \rangle_n, g \in G.$$

This implies that,

$$g.f^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset, g \in G.$$

Therefore, f *is* G *−transitive.*

□

THEOREM 3.5: *Let* X *be a* G *−space, let* $f : X \to X$ *be a function and let* $n \in \mathbb{N}$. *If* $F_n(f)$ *is weakly* G *−mixing, then* f *is weakly* G *−mixing.*

Proof :assume that $F_n(f)$ *is weakly* G *−mixing, we can conclude that* f *is weakly* G *−mixing. Let* $\mathcal{U}_1, \mathcal{U}$, $\mathcal{V}_1$ *and* $\mathcal{V}_2$ *are nonempty open subsets of* X. *Then* $\langle \mathcal{U}_1 \rangle_n$, $\langle \mathcal{U}_2 \rangle_n$, $\langle \mathcal{V}_1 \rangle_n$ *and* $\langle \mathcal{V}_2 \rangle_n$ *are nonempty open subsets of* $F_n(X)$ *and* $(g, h) \in G \times G$. *Since* $F_n(f)$ *is weakly* G *−mixing, there exists* $r \in \mathbb{N}$ *such that,*

$$(g, h).[F_n(f)]^r(\langle \mathcal{U}_i \rangle_n) \cap \langle \mathcal{V}_i \rangle_n \neq \emptyset,$$

for each $i \in \{1, 2\}$ *and* $(g, h) \in G \times G$.

i.e,

$$g.[F_n(f)]^r(\langle \mathcal{U}_1 \rangle_n) \cap \langle \mathcal{V}_1 \rangle_n \neq \emptyset, g \in G$$

and

$$h.[F_n(f)]^r(\langle \mathcal{U}_2 \rangle_n) \cap \langle \mathcal{V}_2 \rangle_n \neq \emptyset, h \in G.$$

Then there exist elements $B_1 \in \langle \mathcal{U}_1 \rangle_n$ *and* $B_2 \in \langle \mathcal{U}_2 \rangle_n$ *such that*

$$g.[F_n(f)]^r(B_1) \in \langle \mathcal{V}_1 \rangle_n, g \in G$$

and

$$h.[F_n(f)]^r(B_2) \in \langle \mathcal{V}_2 \rangle_n, h \in G.$$

Thus, we obtain that

$$g.f^r(B_1) \subseteq \mathcal{V}_1 \text{ and } h.f^r(B_2) \subseteq \mathcal{V}_2, for\ all g, h \in G$$

Hence,

$$g.f^r(\mathcal{U}_1) \cap \mathcal{V}_1 \neq \emptyset, \text{ and } h.f^r(\mathcal{U}_2) \cap \mathcal{V}_2 \neq \emptyset. for\ all g, h \in G$$

Thus,

$$(g, h).(f \times f)^r(\mathcal{U}_1 \times \mathcal{U}_2) \cap (\mathcal{V}_1 \times \mathcal{V}_2) \neq \emptyset, (g, h) \in G \times G.$$

Therefore, f is weakly G −mixing.

□

***THEOREM* 3.6**: Let X be a topological space, $f : X \to X$ be a function and $n \in \mathbb{N}$. If $F_n(X)$ is totally $G-$transitive, then f is totally $G-$transitive.

Proof : Assume that $F_n(f)$ is totally G −transitive, we see that f is totally G −transitive. Let $t \in \mathbb{N}$. We prove that f^s is G −transitive. Let $\mathcal{U}$ and $\mathcal{V}$ be non−empty open subsets of X. Then $\langle\mathcal{U}\rangle_n$ and $\langle\mathcal{V}\rangle_n$ are open subsets of $F_n(X)$ and nonempty. Since $g.F_n(f)$ is totally G −transitive,then $g.[F_n(f)]^s$ is G −transitive. Thus, there is $r \in \mathbb{N}$ such that

$$g.([F_n(f)]^s)^r(\langle\mathcal{U}\rangle_n) \cap \langle\mathcal{V}\rangle_n \neq \emptyset, \text{ for all } g \in G$$

i.e.,

$$g.[F_n(f)]^{sr}(\langle\mathcal{U}\rangle_n) \cap \langle\mathcal{V}\rangle_n \neq \emptyset, g \in G.$$

Then there is $W \in \langle\mathcal{U}\rangle_n$ such that,

$$g.[F_n(f)]^{sr}(W) \in \langle\mathcal{V}\rangle_n, g \in G.$$

Hence,

$$g.f^{sr}(W) \subseteq \mathcal{V}.$$

Thus,

$$g.(f^s)^r(\mathcal{U}) \cap \mathcal{V} \neq \emptyset, g \in G.$$

Thus, f^s is G −transitive. Since s is any positive integer, f is totally G −transitive.

□

***THEOREM* 3.7**: Let X be a topological space, let $f : X \to X$ be a function and let $n \in \mathbb{N}$. If $F_n(f)$ is strongly G −transitive, then f is strongly G −transitive.

Proof : assume that $F_n(f)$ is strongly G −transitive, we show that f is strongly G −transitive. Let $\mathcal{U}$ is open subset of X and nonempty. Then $\langle\mathcal{U}\rangle_n$ is open subset of $F_n(X)$ and nonempty . Thus, according to hypothesis, there is an $s \in \mathbb{N}$ such that:

$$g.\mathrm{F}_n(X) = \bigcup_{k=0}^{s} g.[\mathrm{F}_n(f)]^k(\langle \mathcal{U} \rangle_n),\ g \in G$$

We prove that

$$X = \bigcup_{k=0}^{s} g.f^k(\mathcal{U}),\ g \in G.$$

Let $x \in X$. We prove that

$$\ell \in \bigcup_{k=0}^{s} g.f^k(\mathcal{U}),\ g \in G.$$

Note that $\{\ell\} \in \mathrm{F}_n(X)$. Thus,

$$\{\ell\} \in \bigcup_{k=0}^{s} g.[\mathrm{F}_n(f)]^k(\langle \mathcal{U} \rangle_n),\ g \in G.$$

Then there exists $k_0 \in \{0, \ldots, s\}$ such that

$$\{\ell\} \in g.[\mathrm{F}_n(f)]^{k_0}(\langle \mathcal{U} \rangle_n),\ g \in G.$$

Hence, there exists $A \in \langle U \rangle_n$ such that

$$g.[\mathrm{F}_n(f)]^{k_0}(A) = \{\ell\}\ g \in G.$$

Since $A \subseteq U$,

$$\ell \in \bigcup_{k=0}^{s} g.f^k(\mathcal{U}),\ g \in G.$$

Thus,

$$X \subseteq \bigcup_{k=0}^{s} g.f^k(\mathcal{U}),\ g \in G.$$

Therefore, f is strongly G −transitive.

□

THEOREM 3.8: *Let X be a G −space, let $f : X \to X$ be a function and let $n \in \mathbb{N}$. The set G-$Per(f)$ is a dense subset in X if and only if $G - Per(\mathrm{F}_n(f))$ is a dense subset in $\mathrm{F}_n(X)$.*

THEOREM 3.9: *Let X be a G −space, let $f : X \to X$ be a function and let $n \in \mathbb{N}$. If $\mathrm{F}_n(f)$ is G −chaotic, then f is G −chaotic.*

Proof: We show that f is G −chaotic if we assume that $\mathrm{F}_n(f)$ is G −chaotic. Given that $\mathrm{F}_n(f)$ is G −chaotic, $G - Per(\mathrm{F}_n(f))$ is dense in $\mathrm{F}_n(X)$ and $\mathrm{F}_n(f)$ is G −transitive. Then, by Theorem (3.4), we have that f is G −transitive. Furthermore, by Theorem (3.9), we obtain that $G - Per(f)$ is a dense subset of X. Consequently, f is G −chaotic.

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وقائع مؤتمر البحث العلمي المعاصر ودوره في تحقيق اهداف التنمية المستدامة – تشرين الثاني – 2025 / November

4. conclusion

First, we present some important studies regarding some of the chaotic concepts and characteristics found in literature. Second, in order to prove our theorems, we presented definitions for several chaotic concepts in G-space. Specifically, we study their dynamic properties in G −space between dynamical systems (X,f) and $(\mathcal{F}_n(X), \mathcal{F}_n(f))$*, such as mixing, l.e.o, transitive, weakly mixing, strongly transitive, and chaotic. So, the following is a summary of our work:*

$\mathcal{F}_n(f)$ *G −mixing* $\Longleftrightarrow$ *f G −mixing*
$\mathcal{F}_n(f)$ *G −l.e.o* $\Longleftrightarrow$ *f G −l.e.o*
$\mathcal{F}_n(f)$ *G −transitive* $\longrightarrow$ *f G −transitive*
$\mathcal{F}_n(f)$ *weakly G −mixing* $\longrightarrow$ *f weakly G −mixing*
$\mathcal{F}_n(f)$ *totally G −transitive* $\longrightarrow$ *f totally G −transitive*
$\mathcal{F}_n(f)$ *strongle G −transitive* $\longrightarrow$ *f strongle G −transitive*
$\mathcal{F}_n(f)$ *G −chaotic* $\longrightarrow$ *f G −chaotic*

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